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The nonperturbative closed string tachyon vacuum to high level

Nicolas Moeller

International School for Advanced Studies (SISSA)
via Beirut 2-4, 34014 Trieste, Italy
E-mail: moeller@sissa.it

Haitang Yang

Center for Theoretical Physics, Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A., and
Department of Applied Physics
University of Electronic Science and Technology of China
Chengdu, 610054, People's Republic of China
E-mail: hyanga@mit.edu

Abstract: We compute the action of closed bosonic string field theory at quartic order with fields up to level ten. After level four, the value of the potential at the minimum starts oscillating around a nonzero negative value, in contrast with the proposition made in [5]. We try a different truncation scheme in which the value of the potential converges faster with the level. By extrapolating these values, we are able to give a rather precise value for the depth of the potential.

Keywords: Bosonic Strings, String Field Theory, Tachyon Condensation.

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1. Introduction and summary

In this paper we are addressing the question whether closed bosonic string theory has a stable vacuum. This is of course a non-perturbative problem that needs to be approached in the context of closed string field theory (CSFT) [1]. Its difficulty is two-fold. Firstly, the action of CSFT is non-polynomial in the string field. Secondly, the string field is composed of infinitely many components. As an analytic solution of CSFT seems at present out of reach (even in the light of the newly-discovered solution for the vacuum of open string field theory [2, 3]), we are bound to numerical methods. The first difficulty is probably the most serious but it is believed that truncating the action to a finite power of the string field may furnish a good approximation. The second difficulty is treated by level truncation, keeping in the string field only component fields whose masses are not greater than a given level.

Until recently, only the quadratic and cubic terms of the CSFT action could be computed. A level truncation calculation at this order was done by Kostelecký and Samuel in [4]. They truncated the string field to the massless level, keeping the tachyon, graviton and auxiliary fields, and found a locally stable vacuum with a positive tachyon expectation value. It is now understood [5] that to cubic order we are missing some important interactions, ones that can couple fields whose left-moving and right-moving ghost numbers are not equal. The first scalar field having this property is the ghost-dilaton which plays a central role.

In [6], Belopolsky endeavored the computation of the tachyon effective potential up to quartic order. There were two terms to calculate. Namely the contact term of four tachyons, and the Feynman diagrams with two cubic vertices and four external tachyons. Those terms were combined into one integral over the *whole* (i.e. not reduced) moduli space of spheres with four punctures. Belopolsky then found that this effective potential didn't have any local minimum, the sign and magnitude of the quartic tachyon term were such as to destroy the minimum existing at cubic order. There is however an important flaw in the question of the tachyon effective potential itself. As already mentioned, Yang and Zwiebach have shown in [5], that the zero-momentum ghost dilaton must be included in the tachyon condensate as soon as we are considering quartic terms. As this state is massless, it cannot be integrated out in forming the tachyon effective potential. Instead one should consider the effective potential of the tachyon *and* dilaton.

The computation of the quartic term in the CSFT action was made possible in [7]. This paper solves numerically the geometry of the vertex and gives its solution in terms of fits which can be used to calculate the coupling of any four states. The results of [7] were successfully checked in [8] by verifying the cancellation of the effective coupling of marginal fields to quartic order, and in [9] by checking the cancellation of the effective term with four dilatons.

Yang and Zwiebach then proceeded in [5] to look for a nonperturbative vacuum. This time the dilaton was taken proper care of. They truncated the string field to level four, which included the tachyon (level zero), the dilaton (level two) and four massive fields at level four, and they found a stable vacuum with positive tachyon and dilaton expectation values. The value of the potential at this minimum is negative but seemed to approach zero as the level was increased (and it is also shallower than the vacuum found with the action truncated to cubic order). In the same paper, they studied the low-energy effective action of the tachyon, dilaton and metric, and found that a stable vacuum must have vanishing potential. They went on to propose that this is valid for the full theory, and observed that the numerical results seemed to confirm it. In such low-energy models, a rolling tachyon solution is found. For a large class of potential, the dilaton rolls to positive values corresponding to strong coupling until the universe meets its fate in a big crunch [10]. The natural interpretation of this vacuum would then be that all the degrees of freedom of closed string theory have collapsed, in particular the metric, and thus spacetime, have disappeared. One could then imagine that solitons of CSFT would correspond to spacetimes of lower dimensionality. Some evidence that such solitons exist in CSFT at quartic order was given in [11]. This interpretation is supported by open-closed p-adic string theory [12].

In this paper, we continue the level truncation calculation of [5] and push the computation to level ten. At this level, the string field has a total of 158 fields and the computation of the potential must be automatized. We use the symbolic calculator Mathematica to perform antighost insertions, to calculate correlators (and generate the conservation laws used to calculate them), and to integrate the given results on the reduced moduli space using the results of [7]. The results for the nonperturbative vacuum are not confirming the proposition [5] that its potential should vanish. Instead we see that if we do level truncation in the same way as in [5], the depth of the potential oscillates with the level, and the shallowness at level four is essentially an illusion as the potential takes a dip at level six and then never approaches zero as closely as it did at level four. We then use a different truncation scheme, and find results that are consistent with the former scheme but converge better. This leads us to conclude that CSFT truncated to quartic order has a nonperturbative vacuum with a nonzero potential, given by (3.13).

We conclude this paper by asking how this result would change if we include terms of higher order in the action. In [13], one of us has solved numerically the geometry of the five-point vertex, and checked the result with the dilaton theorem. At this time however only the terms coupling five tachyons or five dilatons have been calculated (other terms will be done in [14]). Although we should really take terms at higher level as well, we are curious and look at how our results change if we include the coupling of five tachyons. As expected from the sign of this term, the potential at the vacuum is pushed towards zero (but is still negative and nonzero). More surprisingly, and perhaps hinting at something important, the oscillations mentioned before are tamed.

The paper is structured as follows: In the rest of this section we briefly summarize how to compute the quartic potential of CSFT. In section 2 we generalize the method of conservation laws to compute correlators on the sphere with four punctures. We describe our results of level truncation in section 3, and finally we include the term with five tachyons and discuss our results in section 4.

We shortly summarize how to calculate quartic multilinear functions, more details can be found in [9, 5]. In our conventions $\alpha' = 2$, and the closed string field theory action is

$$S = -\frac{1}{\kappa^2}\left(\frac{1}{2}\langle\Psi|c_0^- Q_B|\Psi\rangle + \frac{1}{3!}\{\Psi,\Psi,\Psi\} + \frac{1}{4!}\{\Psi,\Psi,\Psi,\Psi\} + \cdots\right), \tag{1.1}$$

where Q_B is the BRST operator, $c_0^{\pm} = \frac{1}{2}(c_0 \pm \bar{c}_0)$, and $\{\ldots\}$ are the multilinear string functions [1]. For the CSFT action to be consistent, the string field $|\Psi\rangle$ must satisfy $(L_0 - \bar{L}_0)|\Psi\rangle = 0$ and $(b_0 - \bar{b}_0)|\Psi\rangle = 0$. We will be working in the Siegel gauge $(b_0 + \bar{b}_0)|\Psi\rangle = 0$. As was shown in [5], the minimal subspace of the Hilbert space for the string field to live in when we are considering tachyon condensation, is the one generated by the scalars obtained by application on the vacuum of Virasoro, ghost and antighost oscillators, and with the additional constraint $\Psi = -\Psi^\star$. The action of $\star$ on a given state changes all left-moving oscillators (Virasoro, ghost and antighost) into right-movers and vice-versa, without changing their orders, and changes the factor in front of the state by its complex conjugate.

To calculate the multilinear function of four states $|\Psi_1\rangle, \ldots, |\Psi_4\rangle$, one inserts them on the sphere at the points $z = 0$, $z = 1$, $z = \infty$ and $z = \xi = x + y\,i$, with an antighost insertion $\mathcal{B}\mathcal{B}^\star$, and one then integrates the corresponding correlator over the reduced moduli space of four-punctured spheres $\mathcal{V}_{0,4}$. It is reduced in the sense that one excludes the spheres that can be obtained as Feynman diagrams built with three-vertices. More explicitly

$$\{\Psi_1, \Psi_2, \Psi_3, \Psi_4\} = \frac{1}{\pi}\int_{\mathcal{V}_{0,4}} dx \wedge dy \langle\Sigma|\mathcal{B}\mathcal{B}^\star|\Psi_1\rangle|\Psi_2\rangle|\Psi_3\rangle|\Psi_4\rangle\,, \tag{1.2}$$

where the antighost insertions are given by ([9])

$$\mathcal{B} = \sum_{I=1}^{4}\sum_{m=-1}^{\infty}\left(B_m^I b_m^{(I)} + \overline{C_m^I}\bar{b}_m^{(I)}\right) \quad , \quad \mathcal{B}^\star = \sum_{I=1}^{4}\sum_{m=-1}^{\infty}\left(C_m^I b_m^{(I)} + \overline{B_m^I}\bar{b}_m^{(I)}\right), \tag{1.3}$$

whose coefficients B^I_m and C^I_m are determined by the four maps from the local coordinates w_I to the uniformizer z

$$B^I_m = \oint \frac{dw}{2\pi i}\frac{1}{w^{m+2}}\frac{1}{h'_I}\frac{\partial h_I}{\partial \xi} \quad , \quad C^I_m = \oint \frac{dw}{2\pi i}\frac{1}{w^{m+2}}\frac{1}{h'_I}\frac{\partial h_I}{\partial \bar{\xi}} \tag{1.4}$$

$$z = h_I(w_I;\xi,\bar{\xi}) = z_I(\xi,\bar{\xi}) + \rho_I(\xi,\bar{\xi})\, w_I + \sum_{n=2}^{\infty} \alpha_{n,I}(\xi,\bar{\xi})\,(\rho_I w_I)^n\,. \tag{1.5}$$

Note that for the puncture at infinity, we should use the coordinate $t = 1/z$ instead of z. Our notation here is a bit different from the notation of [9, 5]. The β_I, γ_I and δ_I used there are related to $\alpha_{m,I}$ by

$$\beta_I \equiv \alpha_{2,I} \quad , \quad \gamma_I \equiv \alpha_{3,I} \quad , \quad \delta_I \equiv \alpha_{4,I}\,. \tag{1.6}$$

The $\alpha_{m,I}$ notation is more convenient at high level because the computation of multilinear functions of fields of level L requires $\alpha_{m,I}$ with $m = 2, \ldots, L/2{+}2$, in our case $m = 2, \ldots, 7$. These coefficients can be deduced from the quadratic differential $\varphi = \phi(z)(dz)^2$ that gives the metric of the interaction worldsheet. Namely it must have poles of second order with residue minus one at the punctures, and its critical graph must be compact (for more details see [16, 17, 6, 7]). For the four-vertex, it is given by

$$\phi(z) = -\frac{(z^2-\xi)^2}{z^2(z-1)^2(z-\xi)^2} + \frac{a(\xi,\bar{\xi})}{z(z-1)(z-\xi)}\,. \tag{1.7}$$

The quadratic differential is thus determined by $a(\xi,\bar{\xi})$, whose solution was constructed numerically in [7]. The expressions of $\alpha_{m,I}$ follow by requiring that in the local coordinates w_I, the quadratic differential takes the form $\phi(w_I) = -1/w_I^2$. All in all the integrand of (1.2) can be expressed as an expression involving ξ, a, $\partial a/\partial\xi$, $\partial a/\partial\bar{\xi}$, and ρ_I, all of which can be directly estimated from the fits given in [7], and correlators on the sphere. Our conventions for these correlators are the same as in [9, 5], namely

$$\langle c(z_1)c(z_2)c(z_3)\bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3)\rangle = -2\langle c(z_1)c(z_2)c(z_3)\rangle_o \cdot \langle \bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3)\rangle_o\,, \tag{1.8}$$

and $\langle c(z_1)c(z_2)c(z_3)\rangle_o = (z_1-z_2)(z_1-z_3)(z_2-z_3)$ is the open string field theory correlator. These will be calculated with the help of the conservation laws described in section 2

The way to do the integration in (1.2) was described in [9]. The whole domain $\mathcal{V}_{0,4}$ can be decomposed into six regions and their complex conjugates, such that

$$\int_{\mathcal{V}_{0,4}} = \int_{\mathcal{A}} + \int_{\frac{1}{\mathcal{A}}} + \int_{1-\mathcal{A}} + \int_{\frac{1}{1-\mathcal{A}}} + \int_{1-\frac{1}{\mathcal{A}}} + \int_{\frac{\mathcal{A}}{\mathcal{A}-1}} + \text{ complex conjugate}\,. \tag{1.9}$$

All of these integrals can be expressed as integrals over $\mathcal{A}$ after pulling back their integrand (see [9] for more details). And at last the two-dimensional region $\mathcal{A}$ was described in [7], so we can do these integrals numerically.

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2. The conservation laws on the spheres with four punctures

As outlined in the previous section, after we let the antighost insertion $\mathcal{B}\mathcal{B}^\star$ act on the states, we must compute correlators of the modified states (by which we mean the external states modified by the antighost insertions). We could do that by performing their conformal transformations from the local coordinates to the sphere. But when the level increases it quickly becomes very tedious to calculate the conformal transformations of the fields given in terms of oscillators acting on the vacuum. We thus need an alternative method for computing correlators; a very convenient one is the method of conservation laws [15]. It was originally constructed to calculate cubic interactions in Witten's cubic string field theory, but it can be generalized to quartic interactions with only notational complications. We review the main idea of this method by considering, as an example, the conservation laws for the ghost $c(z)$. We take a quadratic differential $\phi(z)$, so that the product $\phi(z)c(z)dz$ transforms as a 1-form. And we consider a small contour $\mathcal{C}$ on the sphere, which doesn't encircle any of the punctures 0, 1, ξ and ∞. If $\phi(z)$ is regular everywhere, except possibly at the punctures, the contour can be continuously deformed into the sum of four contours $\mathcal{C}_I$ around each punctures. Expressing each integral in the local coordinates w_I, we thus have

$$0 = \langle\Sigma| \sum_{I=1}^{4} \oint_{\mathcal{C}_I} \phi^{(I)}(w_I) c^{(I)}(w_I) dw_I \,. \tag{2.1}$$

We have $c^{(I)}(w_I) = \sum_n \frac{c_n^{(I)}}{w_I^{n-1}}$, therefore if $\phi^{(I)}(w_I)$ has a pole of order n, the $\mathcal{C}_I$ contour integral will pick up an oscillator c_{2-n} and oscillators with higher indices. We can now explain the method: if we want to get rid of an oscillator $c_{-n}^{(I)}$ at the puncture I, we choose a $\phi(z)$ with a pole of order $2+n$ at the puncture I and poles of lesser order at the other punctures. We can then trade $c_{-n}^{(I)}$ for oscillators $c_m^{(J)}$ with $J = 1, \ldots, 4$ and $m > -n$. Repeating this process, we will eventually be left with only c_1's.

The conservation laws for the Virasoro oscillators are done much in the same way, except for the fact that $T(z)$ is not a tensor if the central charge is not zero. Under a conformal change of variable, it transforms as

$$\tilde{T}(w) = \left(\frac{dz}{dw}\right)^2 T(z) + \frac{c}{12} S(z,w) \,, \tag{2.2}$$

where

$$S(z,w) = \frac{z'''}{z'} - \frac{3}{2}\left(\frac{z''}{z'}\right)^2 \tag{2.3}$$

is the Schwartzian derivative (derivatives are with respect to w), and c is the central charge. Now if $v(z)$ transforms like a vector field, we see that the product $v(z)T(z)dz$ transforms as

$$v(z)T(z)dz = \tilde{v}(w)\left(\tilde{T}(w) - \frac{c}{12}S(z,w)\right) dw \,. \tag{2.4}$$

Repeating the above idea of deforming a small contour, we find the Virasoro conservation laws

$$\langle\Sigma| \sum_{I=1}^{4} \oint_{\mathcal{C}_I} v^{(I)}(w_I)\left(T^{(I)}(w_I) - \frac{c}{12}S(z,w_I)\right) dw_I = 0 \,. \tag{2.5}$$

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Since $b(z)$ has conformal weight two, it transforms as a stress-tensor with zero central charge, we can thus immediately deduce its conservation laws from (2.5).

$$\langle\Sigma| \sum_{I=1}^{4} \oint_{\mathcal{C}_I} v^{(I)}(w_I) b^{(I)}(w_I) dw_I = 0\,. \tag{2.6}$$

2.1 The first conservation laws for $T(z)$

We compute here the first few conservation laws. The higher ones would be too cumbersome to write down, but it will become clear that, like the cubic ones, they can be easily generated on a computer. We start by the conservation laws for $T(z)$, which are slightly easier than $c(z)$ despite the presence of the central charge. Before we begin we must remark that in the case of the cubic vertex, due to its cyclicity, one need only write the conservation laws for one puncture ([15]). For example the conservation law to remove $L^{(1)}_{-n}$ and the one to remove $L^{(2)}_{-n}$ are trivially related by cycling the punctures $I \to I+1 \pmod 3$. For the quartic vertex there is no cyclic symmetry, and we have to write the conservation laws for each of the four punctures.

Since we are considering only descendants of scalar fields with zero momentum, which are annihilated by L_{-1}, we don't need the conservation laws for L_{-1}. Should a L_{-1} appear from another conservation law, we can always commute it away. The first conservation laws are thus the ones for L_{-2}, which we construct now.

We start by expanding the Schwartzian derivative (2.3) in the local coordinates w_I with the definitions (1.5) and (1.6).

$$S(z, w_I) = 6\rho_I^2 \left(\gamma_I - \beta_I^2\right) + \rho_I^3 \left(24\delta_I - 48\beta_I\gamma_I + 24\beta_I^3\right) w_I + \mathcal{O}(w_I^2)\,. \tag{2.7}$$

The Schwartzian derivatives are regular, so they matter only where we have a pole. From the mode expansion

$$T^{(I)}(w_I) = \sum \frac{L_n^{(I)}}{{w_I}^{n+2}}\,, \tag{2.8}$$

we see that we need a vector field $v(z)$ with a pole of order one at the puncture I, and regular everywhere else. In general we will denote $v_{n,I}(z)$ a vector field with expansion in the local coordinates w_I

$$v_{n,I}(w_I) = w_I^{-n+1} + \mathcal{O}(w_I^0)\,, \tag{2.9}$$

and regular everywhere else. It can therefore be used to trade a $L^{(I)}_{-n}$ for oscillators $L^{(J)}_m$, $J = 1, \ldots, 4$, $m \geq -1$. It is easily seen recursively, that we can find such vectors for any $n \geq 2$. Indeed if we have $v_{m,I}(z)$ for $m < n$ and if we write the expansion of the vector field $u(z) = (z - z_I)^{-n+1}$ in the local coordinates w_I as:

$$u(w_I) = \sum_{m=-n+1}^{-1} a_m w_I^m + \mathcal{O}(w_I^0)\,, \tag{2.10}$$

we can take

$$v_{n,I}(z) = \frac{1}{a_{-n+1}} \left(z^{-n+1} - \sum_{m=-n+2}^{-1} a_m v_{1-m,I}(z) \right)\,. \tag{2.11}$$

It will be useful to make the following definitions

$$z_{IJ} \equiv z_I - z_J \,, \qquad s_I \equiv \frac{1}{z_{IJ}} + \frac{1}{z_{IK}} \,, \qquad q_I \equiv \frac{1}{z_{IJ} z_{IK}} \,, \tag{2.12}$$

where the set formed by I, J and K must be $\{1, 2, 3\}$ (regardless of order). We are now ready to calculate the conservation laws. For $L_{-2}^{(I)}$ at the finite punctures $I = 1, 2, 3$, we can take

$$v_{2,I}(z) = \frac{\rho_I^2}{z_{IJ} z_{IK}} \frac{(z - z_J)(z - z_K)}{z - z_I} \,. \tag{2.13}$$

Recalling that the local coordinates w_I are related to the uniformizer z (or $t = 1/z$ for the puncture at infinity) through the conformal maps h_I, given by (1.5) and (1.6) and explicitly rewritten as

$$\begin{aligned} z &= h_I(w_I) = z_I + \rho_I w_I + \rho_I^2 \beta_I w_I^2 + \rho_I^3 \gamma_I w_I^3 + \rho_I^4 \delta_I w_I^4 + \cdots \,, \qquad I = 1, 2, 3 \\ t &= h_4(w_4) = \rho_4 w_4 + \rho_4^2 \beta_4 w_4^2 + \rho_4^3 \gamma_4 w_4^3 + \rho_4^4 \delta_4 w_4^4 + \cdots \,, \end{aligned} \tag{2.14}$$

and using the transformation law of a vector field

$$\tilde{v}(w) = v(z) \frac{dw}{dz} \,, \tag{2.15}$$

we find the following expansions in the local coordinates w_I

$$\begin{aligned} v_{2,I}^{(I)}(w_I) &= \frac{1}{w_I} + \rho_I \left(s_I - 3\beta_I\right) + \rho_I^2 \left(-2\beta_I s_I + q_I + 7\beta_I^2 - 4\gamma_I\right) w_I + \cdots \\ v_{2,I}^{(J)}(w_J) &= \rho_I^2 \frac{z_{JK}}{z_{IJ}^2 z_{KI}} w_J + \cdots \,, \quad J \leq 3 \,, \; J \neq I \\ v_{2,I}^{(4)}(w_4) &= -\frac{\rho_I^2}{z_{IJ} z_{IK}} w_4 + \cdots \,. \end{aligned} \tag{2.16}$$

For the puncture at infinity we take

$$v_{2,4}(t) = \xi \frac{(t-1)\left(t - \frac{1}{\xi}\right)}{t} \rho_4^2 \,, \tag{2.17}$$

which has the expansions

$$\begin{aligned} v_{2,4}^{(4)}(w_4) &= \frac{1}{w_4} - \rho_4 (1 + \xi + 3\beta_4) + \rho_4^2 \left(\xi + 2\beta_4 (1 + \xi) + 7\beta_4^2 - 4\gamma_4\right) w_4 + \cdots \\ v_{2,4}^{(I)}(w_I) &= -\rho_4^2 z_{IJ} z_{IK} \, w_I + \cdots \,, \quad I \leq 3 \,. \end{aligned} \tag{2.18}$$

Now using (2.7), (2.16) and (2.18) in (2.5) we find the conservation laws for L_{-2}

$$\begin{aligned} 0 = \langle \Sigma | &\left(L_{-2} + \frac{c}{2} \rho_I^2 (\beta_I^2 - \gamma_I) + \rho_I \left(s_I - 3\beta_I\right) L_{-1} + \rho_I^2 \left(-2\beta_I s_I + q_I + 7\beta_I^2 - 4\gamma_I\right) L_0 + \cdots \right)^{(I)} \\ &+ \sum_{\substack{J=1 \\ J \neq I}}^{3} \langle \Sigma | \left(\rho_I^2 \frac{z_{JK}}{z_{IJ}^2 z_{KI}} L_0 + \cdots \right)^{(J)} + \langle \Sigma | \left(-\frac{\rho_I^2}{z_{IJ} z_{IK}} L_0 + \cdots \right)^{(4)} \,, \quad I = 1, 2, 3 \end{aligned}$$

$$
\begin{aligned}
0 = &\langle\Sigma| \\
&\times \Big(L_{-2}+\frac{c}{2}\rho_4^2(\beta_4^2-\gamma_4)-\rho_4(1+\xi+3\beta_4)L_{-1}+\rho_4^2\left(\xi+2\beta_4(1+\xi)+7\beta_4^2-4\gamma_4\right)L_0+\cdots\Big)^{(4)} \\
&+\sum_{I=1}^{3}\langle\Sigma|\left(-\rho_4^2 z_{IJ}z_{IK}\,L_0+\cdots\right)^{(I)}\,, \qquad (2.19)
\end{aligned}
$$

where the dots indicate oscillators with indices greater than zero.

Now we go one step further and write the conservations laws for L_{-3}. We are again expanding them up to L_0, so, together with the laws for L_{-2}, they can be used to compute the matter part of all quartic correlators with one field of level six and three other fields of level up to four. We take

$$
\begin{aligned}
v_{3,I}(z) &= \frac{\rho_I^3}{z_{IJ}z_{IK}}\frac{(z-z_J)(z-z_K)}{(z-z_I)^2}-\rho_I(s_I-4\beta_I)\,v_{2,I}(z) \\
v_{3,4}(t) &= \rho_4^3\xi\frac{(t-1)\left(t-\frac{1}{\xi}\right)}{t^2}+\rho_4(1+\xi+4\beta_4)\,v_{2,4}(t)\,, \qquad (2.20)
\end{aligned}
$$

from which we find the conservation laws

$$
\begin{aligned}
0 = &\langle\Sigma|\left(L_{-3}-c\,\rho_I^3\left(2\delta_I-4\beta_I\gamma_I+2\beta_I^3\right)+\rho_I^2\left(-\beta_I^2-5\gamma_I+4\beta_I s_I-s_I^2+q_I\right)L_{-1}\right. \\
&\left.+\rho_I^3\left(2\beta_I^3+12\beta_I\gamma_I-6\delta_I-8\beta_I^2 s_I+2\beta_I q_I+2\beta_I s_I^2-s_I q_I\right)L_0+\cdots\right)^{(I)} \\
&+\sum_{\substack{J=1\\J\neq I}}^{3}\langle\Sigma|\left(\frac{\rho_I^3 z_{JK}}{Z_{IJ}^2 z_{IK}}\left(\frac{1}{z_{IJ}}+s_I-4\beta_I\right)L_0+\ldots\right)^{(J)}+\langle\Sigma|\left(\frac{\rho_I^3}{z_{IJ}z_{IK}}(s_I-4\beta_I)\,L_0+\ldots\right)^{(4)} \\
0 = &\langle\Sigma|\left(L_{-3}-c\,\rho_4^3\left(2\delta_4-4\beta_4\gamma_4+2\beta_4^3\right)-\rho_4^2\left(\beta_4^2+4(1+\xi)\beta_4+5\gamma_4+1+\xi+\xi^2\right)L_{-1}\right. \\
&\left.+\rho_4^3\left(2\beta_4^3+8(1+\xi)\beta_4^2+2(1+3\xi+\xi^2)\beta_4+12\beta_4\gamma_4-6\delta_4+\xi+\xi^2\right)L_0+\cdots\right)^{(4)} \\
&+\sum_{I=1}^{3}\langle\Sigma|\left(-\rho_4^3\left(z_I^2(1-\xi)(-1)^I+z_{IJ}z_{IK}(1+\xi+4\beta_4)\right)L_0+\cdots\right)^{(I)}\,. \qquad (2.21)
\end{aligned}
$$

We emphasize again that the conservation laws for b_{-n} are the same as for L_{-n} after setting the central charge c to zero.

2.2 The first conservation laws for $c(z)$

If the string states are in the Siegel gauge, they will carry no c_0 oscillators, so we don't need the conservation laws for c_0. One may worry that a $c_0^{(I)}$ may arise from a term w_I^{-2} in another conservation law, but we can avoid this because we can always remove such a term by subtracting multiples of the quadratic differentials given by

$$
\begin{aligned}
\phi_{0,I}(z) &= \frac{z_{IJ}z_{IK}}{(z-z_I)^2(z-z_J)(z-z_K)}, \quad I=1,2,3 \\
\phi_{0,4}(t) &= \frac{\xi^{-1}}{t^2(t-1)(t-\xi^{-1})}\,. \qquad (2.22)
\end{aligned}
$$

We see that $\phi_{0,I}(z)$ has a pole of order 2 with unit coefficient at the puncture z_I, and poles of order one at two other punctures. For $I<4$, $\phi_{0,I}(z)$ is finite at infinity. We denote by

$\phi_{n,I}(z)$ a quadratic differential with expansion in the local coordinates w_I

$$\phi_{n,I}(w_I) = w_I^{-n-2} + \mathcal{O}(w_I^{-1})\,, \tag{2.23}$$

and regular everywhere expect for possible poles of order one at other punctures. It can therefore be used to trade a $c_{-n}^{(I)}$ for oscillators $c_m^{(J)}$, $J = 1\ldots,4$, $m \geq 1$. Again, it is easy to see that we can find such quadratic differentials for any $n \geq 1$.

We can now write the conservation laws for c_{-1}. For the finite punctures $I = 1, 2, 3$, we can take

$$\phi_{1,I}(z) = \left(\frac{\rho_I}{z - z_I} - \rho_I(\beta_I - s_I)\right)\phi_{0,I}(z)\,. \tag{2.24}$$

Using the transformation law of a quadratic differential $\phi(z)$

$$\tilde{\phi}(w)dw^2 = \phi(z)dz^2\,, \tag{2.25}$$

and the conformal maps (2.14), we can write the expansions of $\phi_{1,I}(z)$ in the local coordinates

$$\begin{aligned}
\phi_{1,I}^{(I)}(w_I) &= \frac{1}{w_I^3} + \rho_I^2\left(-4\beta_I^2 + \beta_I s_I + 3\gamma_I - q_I\right)\frac{1}{w_I} + \cdots \\
\phi_{1,I}^{(J)}(w_J) &= -\frac{\rho_I\rho_J z_{IK}}{z_{IJ}z_{JK}}\left(\frac{1}{z_{IJ}} + \beta_I - s_I\right)\frac{1}{w_J} + \cdots \\
\phi_{1,I}^{(4)}(w_4) &= \mathcal{O}(w_4^0)\,.
\end{aligned} \tag{2.26}$$

For the puncture at infinity we take

$$\phi_{1,4}(t) = \frac{\rho_4}{t^3} - \beta_4\rho_4\phi_{0,4}(t)\,, \tag{2.27}$$

which has the expansions

$$\begin{aligned}
\phi_{1,4}^{(4)}(w_4) &= \frac{1}{w_4^3} + \rho_4^2\left(3\gamma_4 - 4\beta_4^2 - (1+\xi)\beta_4\right)\frac{1}{w_4} + \cdots \\
\phi_{1,4}^{(I)}(w_I) &= \rho_4\rho_I\left(\delta_{I1} - (-1)^I(1-\delta_{I1})\frac{\beta_4}{1-\xi}\right)\frac{1}{w_I} + \cdots\,.
\end{aligned} \tag{2.28}$$

From these expansions we deduce the conservation laws for c_{-1}

$$\begin{aligned}
0 &= \langle\Sigma|\left(c_{-1} + \rho_I^2\left(-4\beta_I^2 + \beta_I s_I + 3\gamma_I - q_I\right)c_1 + \cdots\right)^{(I)} \\
&\quad + \sum_{\substack{J=1\\J\neq I}}^{3}\langle\Sigma|\left(-\frac{\rho_I\rho_J z_{IK}}{z_{IJ}z_{JK}}\left(\frac{1}{z_{IJ}} + \beta_I - s_I\right)c_1 + \cdots\right)^{(J)} + \langle\Sigma|\left(\ldots\right)^{(4)} \\
0 &= \langle\Sigma|\left(c_{-1} + \rho_4^2\left(3\gamma_4 - 4\beta_4^2 - (1+\xi)\beta_4\right)c_1 + \cdots\right)^{(4)} \\
&\quad + \sum_{I=1}^{3}\langle\Sigma|\left(\rho_4\rho_I\left(\delta_{I1} - (-1)^I(1-\delta_{I1})\frac{\beta_4}{1-\xi}\right)c_1 + \cdots\right)^{(I)}\,.
\end{aligned} \tag{2.29}$$

We now write the next conservation laws, for c_{-2}. For the vector fields, we take

$$\phi_{2,I}(z) = \frac{\rho_I^2}{(z - z_I)^4} + 2\rho_I^2(\beta_I^2 - \gamma_I)\,\phi_{0,I}(z)$$

$$\phi_{2,4}(t) = \frac{\rho_4^2}{t^4} + 2\rho_4^2(\beta_4^2 - \gamma_4)\,\phi_{0,4}(t)\,. \tag{2.30}$$

And we find

$$\begin{aligned}
0 &= \langle\Sigma|\left(c_{-2} + 2\rho_I^3\left(4\beta_I^3 - 6\beta_I\gamma_I + 2\delta_I + (\gamma_I - \beta_I^2)s_I\right)\,c_1 + \cdots\right)^{(I)} \\
&+ \sum_{\substack{J=1\\ J\neq I}}^{3}\langle\Sigma|\left(2\rho_I^2\rho_J(\beta_I^2 - \gamma_I)\frac{z_{IK}}{z_{IJ}z_{JK}}\,c_1 + \cdots\right)^{(J)} + \langle\Sigma|\,(\ldots)^{(4)} \\
0 &= \langle\Sigma|\left(c_{-2} + 2\rho_4^3\left(4\beta_4^3 - 6\beta_4\gamma_4 + 2\delta_4 - (1+\xi)(\gamma_4 - \beta_4^2)\right)\,c_1 + \cdots\right)^{(4)} \\
&+ \sum_{I=1}^{3}\langle\Sigma|\left(2(-1)^I\rho_4^2\rho_I(1-\delta_{I1})\frac{\beta_4^2 - \gamma_4}{1-\xi}\,c_1 + \cdots\right)^{(I)}\,.
\end{aligned} \tag{2.31}$$

2.3 An example

We want here to give a simple but nontrivial example of a quartic correlator computation that uses some of the above conservation laws. Let us take one field of level four and one field of level six (see section 3 for the list of fields and their notation). We choose

$$\begin{aligned}
|\Psi_4\rangle &= c_{-1}\bar{c}_{-1}|0\rangle \\
|\Psi_{12}\rangle &= c_{-2}\bar{c}_{-2}|0\rangle\,.
\end{aligned} \tag{2.32}$$

And we want to calculate the quartic amplitude of Ψ_4, Ψ_{12}, and two tachyons.

$$\{T, \Psi_4, \Psi_{12}, T\} = \frac{1}{\pi}\int_{\mathcal{V}_{0,4}} dx\wedge dy\langle\Sigma|\mathcal{B}\mathcal{B}^\star|T\rangle|\Psi_4\rangle|\Psi_{12}\rangle|T\rangle\,. \tag{2.33}$$

where the antighost insertions are given by (1.3) and (1.4). We find

$$\begin{aligned}
\langle\Sigma|\mathcal{B}\mathcal{B}^\star|T\rangle|\Psi_4\rangle|\Psi_{12}\rangle|T\rangle = &-2\left(B_1^2\bar{B}_1^2 - C_1^2\bar{C}_1^2\right)\,\langle c_1, 1, c_{-2}, c_1\rangle_o\langle\bar{c}_1, 1, \bar{c}_{-2}, \bar{c}_1\rangle_o \\
&-2\left(B_2^3\bar{B}_2^3 - C_2^3\bar{C}_2^3\right)\,\langle c_1, c_{-1}, 1, c_1\rangle_o\langle\bar{c}_1, \bar{c}_{-1}, 1, \bar{c}_1\rangle_o \\
&+2\left(B_1^2\bar{B}_2^3 - C_1^2\bar{C}_2^3\right)\,\langle c_1, 1, c_{-2}, c_1\rangle_o\langle\bar{c}_1, \bar{c}_{-1}, 1, \bar{c}_1\rangle_o \\
&+2\left(B_2^3\bar{B}_1^2 - C_2^3\bar{C}_1^2\right)\,\langle c_1, c_{-1}, 1, c_1\rangle_o\langle\bar{c}_1, 1, \bar{c}_{-2}, \bar{c}_1\rangle_o\,.
\end{aligned} \tag{2.34}$$

We therefore need to compute the two open correlators $\langle c_1, c_{-1}, 1, c_1\rangle_o$ and $\langle c_1, 1, c_{-2}, c_1\rangle_o$, on the four-punctured sphere Σ. To calculate the first one, we use the conservation laws (2.29) to exchange the c_{-1} on the second puncture for a c_1 on the second puncture and a c_1 on the third puncture. Namely

$$\begin{aligned}
\langle c_1, c_{-1}, 1, c_1\rangle_o &= -\rho_2^2\left(-4\beta_2^2 + \beta_2 s_2 + 3\gamma_2 - q_2\right)\,\langle c_1, c_1, 1, c_1\rangle_o \\
&\quad+\rho_2\rho_3\frac{1}{\xi(1-\xi)}\left(\frac{1}{1-\xi} + \beta_2 - s_2\right)\,\langle c_1, 1, c_1, c_1\rangle_o \\
&= \frac{\rho_2}{\rho_1\rho_4}\left(4\beta_2^2 - \beta_2 - 3\gamma_2\right)\,.
\end{aligned} \tag{2.35}$$

Similarly, we use the conservation laws for c_{-2} (2.31) to compute the second correlator by exchanging the c_{-2} on the third puncture for a c_1 on the third puncture and a c_1 on the second puncture. We find

$$\begin{aligned}\langle c_1, 1, c_{-2}, c_1\rangle_o &= -2\rho_3^3\left(4\beta_3^3 - 6\beta_3\gamma_3 + 2\delta_3 + (\gamma_3 - \beta_3^2)s_3\right)\langle c_1, 1, c_1, c_1\rangle_o \\ &\quad -2\rho_3^2\rho_2\left(\beta_3^2 - \gamma_3\right)\frac{\xi}{\xi - 1}\langle c_1, c_1, 1, c_1\rangle_o \\ &= -2\frac{\rho_3^2}{\rho_1\rho_4}\left(\xi\left(4\beta_3^2 - 6\beta_3\gamma_3 + 2\delta_3\right) + \gamma_3 - \beta_3^2\right) . \end{aligned} \tag{2.36}$$

The integral in (2.33) can then by expressed as an integral on the region $\mathcal{A}$ (see (1.9)) as explained in [9], and the numerical integration on $\mathcal{A}$ can be done by using the fits given in [7].

3. The results

The string field

We start this section by writing the components of the string field. We recall the string field up to level four, and compare our notation with the one in [5]. Then we list all the fields at level six. For level eight and ten, we describe a simple way to write down all the closed fields from open fields of all ghost numbers.

We will write the string field in terms of components ψ_i depending on one index.

$$|\Psi\rangle = \sum_{i\geq 1}\psi_i|\Psi_i\rangle . \tag{3.1}$$

The first field $|\Psi_1\rangle$ is the only field of level zero, namely the tachyon

$$|\Psi_1\rangle = c_1\bar{c}_1|0\rangle . \tag{3.2}$$

Then $|\Psi_2\rangle$ is the field of level two, the dilaton

$$|\Psi_2\rangle = \left(c_1c_{-1} - \bar{c}_1\bar{c}_{-1}\right)|0\rangle . \tag{3.3}$$

Before going further, it is good to introduce a way of listing the closed fields in a relatively simple manner. The elementary closed fields $|\Psi_k\rangle$ can be written

$$|\Psi_k\rangle = \left(\mathcal{O}_{k_1}\mathcal{O}^\star_{k_2} - \mathcal{O}^\star_{k_1}\mathcal{O}_{k_2}\right)|0\rangle , \tag{3.4}$$

where $\mathcal{O}_{k_{1,2}}$ are products of left-moving oscillators. The $\star$ conjugation was defined in [5] on closed fields, here it simply changes all left-moving oscillators to right-moving oscillators without changing their order. Note that the expression (3.4) is invariant under world-sheet parity $\mathcal{P}$, whose action is

$$\mathcal{P}\Psi = -\Psi^\star . \tag{3.5}$$

Indeed, it was shown in [5] that we may consistently restrict the string field to have $\mathcal{P}$-eigenvalue one. Let us look at the open string states $\mathcal{O}_{k_1}|0\rangle$ and $\mathcal{O}_{k_2}|0\rangle$. Because the closed

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L	G	open string states $\lvert L,G,i\rangle$, $i=1,\ldots,n_{L,G}$	$n_{L,G}$
0	1	$c_1\lvert 0\rangle$	1
1	0	$\lvert 0\rangle$	1
	2	$c_{-1}c_1\lvert 0\rangle$	1
2	0	$b_{-2}c_1\lvert 0\rangle$	1
	1	$c_{-1}\lvert 0\rangle,\ \ L_{-2}c_1\lvert 0\rangle$	2
	2	$c_{-2}c_1\lvert 0\rangle$	1
3	-1	$b_{-2}\lvert 0\rangle$	1
	0	$L_{-2}\lvert 0\rangle,\ \ b_{-3}c_1\lvert 0\rangle$	2
	1	$c_{-2}\lvert 0\rangle,\ \ L_{-3}c_1\lvert 0\rangle,\ \ b_{-2}c_{-1}c_1\lvert 0\rangle$	3
	2	$c_{-3}c_1\lvert 0\rangle,\ \ L_{-2}c_{-1}c_1\lvert 0\rangle$	2
	3	$c_{-2}c_{-1}c_1\lvert 0\rangle$	1

Table 1: The open string fields of level L and ghost number G for levels 0 to 3.

string state must satisfy $\left(L_0-\bar{L}_0\right)\lvert\Psi_k\rangle=0$, these two open string states must have the same level L. Moreover, their ghost numbers must add to two. If we write an open string state of level L and ghost number G as $\lvert L,G,i\rangle$, where i is an index running from one to the number $n_{L,G}$ of such open states, we can write

$$\lvert\Psi_k\rangle=\lvert L_k,G_k,i_k\rangle\otimes\lvert L_k,2-G_k,j_k\rangle^\star-\lvert L_k,G_k,i_k\rangle^\star\otimes\lvert L_k,2-G_k,j_k\rangle\,. \tag{3.6}$$

The definition of the $\star$-conjugation has been trivially extended here, its action on a left-moving open string state is a right-moving open-string state. We list the open string states $\lvert L,G,i\rangle$ in table 1 for $L=0,1,2,3$ and in table 3 for $L=4,5$.

Given these tables, it is now straightforward to write down all closed fields at level L. As a preliminary we see from the construction (3.6) and from the fact that $n_{L,G}=n_{L,2-G}$, that the number N_L of closed string states at level L is

$$N_L=\sum_{G=2}^{\infty}n_{L/2,G}^2+\frac{1}{2}n_{L/2,1}\left(n_{L/2,1}+1\right)\,. \tag{3.7}$$

We list in table 2, the numbers N_L for L up to 24. In this paper we shall limit ourselves to level 10, the computational limit of our codes.

We can now continue to list the closed states. At level four, we read from table 1

$$\lvert\Psi_3\rangle\;=\;\left(b_{-2}c_1\bar{c}_{-2}\bar{c}_1-\bar{b}_{-2}\bar{c}_1c_{-2}c_1\right)\lvert 0\rangle$$

L	0	2	4	6	8	10	12	14	16	18	20	22	24
N_L	1	1	4	11	38	103	314	807	2148	5282	12872	29792	68526

Table 2: The numbers of closed string states N_L at level L.

$$\begin{aligned}
|\Psi_4\rangle &= c_{-1}\bar{c}_{-1}|0\rangle \\
|\Psi_5\rangle &= L_{-2}c_1\bar{L}_{-2}\bar{c}_1|0\rangle \\
|\Psi_6\rangle &= \left(c_{-1}\bar{L}_{-2}\bar{c}_1 - \bar{c}_{-1}L_{-2}c_1\right)|0\rangle\,.
\end{aligned}$$

So our fields ψ_i up to level 4 are related to the fields of [5] by

$$\psi_1 = t\,,\quad \psi_2 = d\,,\quad \psi_3 = g_1\,,\quad \psi_4 = f_1\,,\quad \psi_5 = f_2\,,\quad \psi_6 = f_3\,. \tag{3.8}$$

In order to facilitate comparisons, we will keep the names t, d, g_1, f_1, f_2 and f_3 for these fields. At level six, we have

$$\begin{aligned}
|\Psi_7\rangle &= \left(b_{-2}\bar{c}_{-2}\bar{c}_{-1}\bar{c}_1 - \bar{b}_{-2}c_{-2}c_{-1}c_1\right)|0\rangle \\
|\Psi_8\rangle &= \left(L_{-2}\bar{c}_{-3}\bar{c}_1 - \bar{L}_{-2}c_{-3}c_1\right)|0\rangle \\
|\Psi_9\rangle &= L_{-2}\bar{L}_{-2}\left(\bar{c}_{-1}\bar{c}_1 - c_{-1}c_1\right)|0\rangle \\
|\Psi_{10}\rangle &= \left(b_{-3}c_1\bar{c}_{-3}\bar{c}_1 - \bar{b}_{-3}\bar{c}_1c_{-3}c_1\right)|0\rangle \\
|\Psi_{11}\rangle &= \left(b_{-3}c_1\bar{L}_{-2}\bar{c}_{-1}\bar{c}_1 - \bar{b}_{-3}\bar{c}_1L_{-2}c_{-1}c_1\right)|0\rangle \\
|\Psi_{12}\rangle &= c_{-2}\bar{c}_{-2}|0\rangle \\
|\Psi_{13}\rangle &= L_{-3}c_1\bar{L}_{-3}\bar{c}_1|0\rangle \\
|\Psi_{14}\rangle &= b_{-2}c_{-1}c_1\bar{b}_{-2}\bar{c}_{-1}\bar{c}_1|0\rangle \\
|\Psi_{15}\rangle &= \left(c_{-2}\bar{L}_{-3}\bar{c}_1 - \bar{c}_{-2}L_{-3}c_1\right)|0\rangle \\
|\Psi_{16}\rangle &= \left(c_{-2}\bar{b}_{-2}\bar{c}_{-1}\bar{c}_1 - \bar{c}_{-2}b_{-2}c_{-1}c_1\right)|0\rangle \\
|\Psi_{17}\rangle &= \left(L_{-3}c_1\bar{b}_{-2}\bar{c}_{-1}\bar{c}_1 - \bar{L}_{-3}\bar{c}_1b_{-2}c_{-1}c_1\right)|0\rangle\,.
\end{aligned}$$

For levels 8 and 10, we don't explicitly write the $38+103$ fields, but we refer to table 3, and we specify in which order we do the constructions (3.6). First we do $G=-\infty,\ldots,0$, $i=1,\ldots,n_{L/2,G}$, $j=1,\ldots,n_{L/2,G}$. Then $G=1$, $i=j=1,\ldots,n_{L/2,G}$. And finally $G=1$, $i=1,\ldots,n_{L/2,G}$, $j=i+1,\ldots,n_{L/2,G}$.

The vacuum

We will consider two different truncation schemes A and B. In the scheme A (which was used in [5]), we keep all the fields up to some fixed level L (which in this paper will be $L=10$), and we progressively increase the interaction level M of the quartic potential, $M=0,2,,\ldots,10$. In the scheme B we progressively increase the maximal fields level L (here $L=2$ to $L=10$ and we do not consider fields of level higher than L), and for each L we take the full quartic potential, i.e. the one with interaction level $M=4L$ (this is similar to what is usually done in cubic string field theory).

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L	G	open string states $\vert L, G, i\rangle$, $i = 1, \ldots, n_{L,G}$	$n_{L,G}$
4	-1	$b_{-3}\vert 0\rangle$	1
	0	$L_{-3}\vert 0\rangle,\ L_{-2}b_{-2}c_1\vert 0\rangle,\ b_{-4}c_1\vert 0\rangle,\ b_{-2}c_{-1}\vert 0\rangle$	4
	1	$L_{-4}c_1\vert 0\rangle,\ L_{-2}c_{-1}\vert 0\rangle,\ L_{-2}L_{-2}c_1\vert 0\rangle,\ c_{-3}\vert 0\rangle,\ b_{-3}c_{-1}c_1\vert 0\rangle,\ b_{-2}c_{-2}c_1\vert 0\rangle$	6
	2	$L_{-2}c_{-2}c_1\vert 0\rangle,\ L_{-3}c_{-1}c_1\vert 0\rangle,\ c_{-4}c_1\vert 0\rangle,\ c_{-2}c_{-1}\vert 0\rangle$	4
	3	$c_{-3}c_{-1}c_1\vert 0\rangle$	1
5	-1	$b_{-4}\vert 0\rangle,\ L_{-2}b_{-2}\vert 0\rangle,\ b_{-3}b_{-2}c_1\vert 0\rangle$	3
	0	$L_{-4}\vert 0\rangle, L_{-3}b_{-2}c_1\vert 0\rangle, L_{-2}L_{-2}\vert 0\rangle, L_{-2}b_{-3}c_1\vert 0\rangle, b_{-5}c_1\vert 0\rangle, b_{-3}c_{-1}\vert 0\rangle, b_{-2}c_{-2}\vert 0\rangle$	7
	1	$L_{-5}c_1\vert 0\rangle,\ L_{-3}c_{-1}\vert 0\rangle,\ L_{-3}L_{-2}c_1\vert 0\rangle,\ L_{-2}c_{-2}\vert 0\rangle,\ L_{-2}b_{-2}c_{-1}c_1\vert 0\rangle,\ c_{-4}\vert 0\rangle,$ $b_{-4}c_{-1}c_1\vert 0\rangle,\ b_{-3}c_{-2}c_1\vert 0\rangle,\ b_{-2}c_{-3}c_1\vert 0\rangle$	9
	2	$L_{-4}c_{-1}c_1\vert 0\rangle,\ L_{-3}c_{-2}c_1\vert 0\rangle,\ L_{-2}c_{-3}c_1\vert 0\rangle,\ L_{-2}L_{-2}c_{-1}c_1\vert 0\rangle,\ c_{-5}c_1\vert 0\rangle,$ $c_{-3}c_{-1}\vert 0\rangle,\ b_{-2}c_{-2}c_{-1}c_1\vert 0\rangle$	7
	3	$L_{-2}c_{-2}c_{-1}c_1\vert 0\rangle,\ c_{-4}c_{-1}c_1\vert 0\rangle,\ c_{-3}c_{-2}c_1\vert 0\rangle$	3

Table 3: The open string fields of level L and ghost number G for levels 4 and 5.

We start by giving the relevant quartic potentials that we computed. The quadratic and cubic potentials with fields up to level six are written down in appendix A. For the truncation scheme B, we need to extend the notations of [5]. We define $V^{(4)}_{L,M}$ to be the quartic potential at level M only, with fields of level up to L. We note that if $L > M$, we have $V^{(4)}_{L,M} = V^{(4)}_{M,M}$. We then define the total potential to level M with fields to level L

$$\mathbb{V}^{(4)}_{L,M} \equiv \mathbb{V}^{(3)}_{L,3L} + \sum_{i=0}^{M/2} V^{(4)}_{L,2i}\,, \tag{3.9}$$

where $\mathbb{V}^{(3)}_{L,3L}$ is the complete quadratic and cubic potential with fields up to level L. We note that, at the highest level that we are considering, $L = 10$, we take $\mathbb{V}^{(3)}_{10,24}$ instead of $\mathbb{V}^{(3)}_{10,30}$. Indeed this last potential is too big for our symbolic calculator, but we emphasize that the difference in the results is minute, as can be verified by comparing the results using, for example, $\mathbb{V}^{(3)}_{8,20}$ and $\mathbb{V}^{(3)}_{8,24}$. Scheme B would require that we compute all potentials up to $V^{(4)}_{10,40}$, but this computation would be impossible within a reasonable time with our codes on a desktop computer. We are able to compute $V^{(4)}_{0,0}$, $V^{(4)}_{2,2}$, $V^{(4)}_{4,4}$, $V^{(4)}_{6,6}$, $V^{(4)}_{8,8}$, $V^{(4)}_{10,10}$, $V^{(4)}_{10,12}$, $V^{(4)}_{6,14}$, $V^{(4)}_{6,16}$. We will see below that these potentials are already enough to give a good picture of scheme B. Of course if $L' < L$, the potential $V^{(4)}_{L',M}$ can be obtained from $V^{(4)}_{L,M}$ simply by deleting the terms with fields of level greater than L'. The quadratic and cubic potentials with fields up to level six are shown in appendix A. Here are some of the

Potential	t	d	f_1	f_2	f_3	g_1	Value of the potential
$\mathbb{V}^{(3)}_{10,24}$	0.4392	0	−0.06836	−0.009648	−0.02748	0	−0.06394
$\mathbb{V}^{(4)}_{10,0}$	−−	−−	−−	−−	−−	−−	−−
$\mathbb{V}^{(4)}_{10,2}$	0.3182	0.4955	−0.08272	−0.006138	−0.02679	−0.1039	−0.05429
$\mathbb{V}^{(4)}_{10,4}$	0.2311	0.4638	−0.04815	−0.001680	−0.01338	−0.07412	−0.03207
$\mathbb{V}^{(4)}_{10,6}$	0.4016	0.4261	−0.1457	−0.008684	−0.04016	−0.03602	−0.06860
$\mathbb{V}^{(4)}_{10,8}$	0.3194	0.4268	−0.1322	−0.01145	−0.04284	−0.1051	−0.05368
$\mathbb{V}^{(4)}_{10,10}$	0.2901	0.4587	−0.1046	−0.007365	−0.03376	−0.1095	−0.04933

Table 4: The value of the potential and the expectation values of the first few fields at the nonperturbative vacuum in the truncation scheme A.

aforementioned quartic potentials

$$\begin{aligned}
\kappa^2 V^{(4)}_{0,0} &= -3.017\, t^4 \\
\kappa^2 V^{(4)}_{2,2} &= 3.872\, t^3 d \\
\kappa^2 V^{(4)}_{4,4} &= 1.368\, d^2 t^2 - 0.4377\, f_1 t^3 - 56.26\, f_2 t^3 + 13.02\, f_3 t^3 + 0.2725\, g_1 t^3 \\
\kappa^2 V^{(4)}_{6,6} &= -0.9528\, t d^3 + t^2 d\,(5.049\, g_1 + 2.385\, f_1 + 49.09\, f_2 - 20.14\, f_3) \\
&\quad + t^3\,(1.678\,\psi_8 + 16.36\,\psi_9 + 0.5357\,\psi_{10} + 5.034\,\psi_{11} - 0.1790\,\psi_{12} \\
&\qquad -91.70\,\psi_{13} - 0.7159\,\psi_{14} + 8.255\,\psi_{15} + 0.7159\,\psi_{16} - 16.51\,\psi_{17}) \\
\kappa^2 V^{(4)}_{8,8} &= -0.1056\, d^4 + t d^2\,(-3.226\, g_1 + 0.2779\, f_1 + 19.31\, f_2 - 5.047\, f_3) \\
&\quad + t^2 d\,(1.043\,\psi_7 - 2.393\,\psi_8 + 19.31\,\psi_9 + 1.325\,\psi_{10} - 7.180\,\psi_{11} + 0.3375\,\psi_{12} \\
&\qquad +98.84\,\psi_{13} + 1.350\,\psi_{14} - 12.69\,\psi_{15} - 2.393\,\psi_{16} + 25.38\,\psi_{17}) \\
&\quad + t^2\,\big(3.816\, g_1^2 + 0.6519\, g_1 f_1 + 3.429\, g_1 f_2 + 1.025\, g_1 f_3 + 0.2566\, f_1^2 \\
&\qquad -10.98\, f_1 f_2 - 1.906\, f_1 f_3 - 979.3\, f_2^2 + 314.4\, f_2 f_3 - 10.59\, f_3^2\big) \\
&\quad + t^3\,(-1.872\,\psi_{19} + 32.94\,\psi_{20} + 0.7143\,\psi_{21} + 1.711\,\psi_{22} + 1.143\,\psi_{23} - 3.750\,\psi_{24} \\
&\qquad +0.09003\,\psi_{25} - 0.3521\,\psi_{26} + 0.1803\,\psi_{27} + 2.854\,\psi_{28} + 0.1263\,\psi_{29} + 0.09024\,\psi_{30} \\
&\qquad -0.3518\,\psi_{31} + 422.0\,\psi_{32} + 0.0452\,\psi_{33} + 0.2043\,\psi_{34} - 212.9\,\psi_{35} - 3.660\,\psi_{36} \\
&\qquad -831.3\,\psi_{37} - 0.04596\,\psi_{38} - 0.4136\,\psi_{39} - 0.1758\,\psi_{40} + 39.28\,\psi_{41} - 658.1\,\psi_{42} \\
&\qquad +7.068\,\psi_{43} - 21.20\,\psi_{44} - 9.795\,\psi_{45} + 123.6\,\psi_{46} - 0.3480\,\psi_{47} + 1.044\,\psi_{48} \\
&\qquad +0.01764\,\psi_{49} + 10.34\,\psi_{50} - 31.01\,\psi_{51} - 5.997\,\psi_{52} + 0.2757\,\psi_{53} + 0.1697\,\psi_{54} \\
&\qquad -0.5091\,\psi_{55})\,.
\end{aligned} \tag{3.10}$$

The numerical coefficients are rounded to four significant digits, corresponding to the precision that the fit of the quartic geometry [7] allows to reach.

Scheme A

In table 4 we show our results for the nonperturbative minimum of the potential in the truncation scheme A. We also give the vacuum expectation values of the tachyon, dilaton and fields of level four. The lines up to interaction level four are very similar to the results of [5], the small differences coming from the quadratic and cubic interactions with fields of level higher than four; these are clearly unimportant contributions and the results agree qualitatively. Looking at the value of the potential, we see that although, up to level four, it seemed to approach monotonically zero, it is actually oscillating around a value of about -0.05. This oscillation, which is also visible on the expectation values of the fields, is quite strong and makes it difficult to draw an accurate conclusion from this data.

Scheme B

Here we want to look at the minimum of $\mathbb{V}^{(4)}_{L,4L}$ for $L = 2, \ldots, 10$. As we have already said, we can't fully compute these potentials for $L > 4$. To remedy this, we are going to look at the values of the potential at the minimum of $\mathbb{V}^{(4)}_{L,M}$ for fixed L and all M starting at two and up as far as we can. This data is shown in the columns of table 5. Looking at the

M	$L=2$	$L=4$	$L=6$	$L=8$	$L=10$
2	-0.1002	-0.05806	-0.05822	-0.05422	-0.05429
4	-0.05071	-0.03383	-0.03402	-0.03199	-0.03207
6	-0.08141	-0.07194	-0.07204	-0.06850	-0.06860
8	-0.08534	-0.05834	-0.05674	-0.05367	-0.05368
10	$--$	-0.05178	-0.05181	-0.04928	-0.04933
12	$--$	-0.05509	-0.05516	-0.05210	-0.05193
14	$--$	-0.05437	-0.05427	$--$	$--$
16	$--$	-0.05442	-0.05438	$--$	$--$
$4L$	-0.0853	-0.0544	-0.0544	-0.0514	-0.0513

Table 5: The values of the potentials $\kappa^2 V_{L,M}$ at the vacuum, and the extrapolation of the value of $\kappa^2 V_{L,4L}$.

longest complete data that we have, namely $L = 4$, we see that the value of the potential at the vacuum oscillates (except from $M = 6$ to $M = 10$ where it always increases when $L \geq 4$) and converges relatively fast. We are thus making the assumption that the final result is always between the last two available values and closer to the last one, namely

$$\kappa^2 \mathbb{V}^{(4)}_{L,4L} \approx \alpha\, \kappa^2 \mathbb{V}^{(4)}_{L,Q} + (1-\alpha)\, \kappa^2 \mathbb{V}^{(4)}_{L,Q+2}\,, \tag{3.11}$$

with $0 < \alpha < 0.5$. And we are making the further assumption that α doesn't depend much on Q and L. Once α is estimated, we should use the larger Q possible in order to have an accurate extrapolation. The value of α that would give the right answer for $L = 4$ (with $Q = 10$) is approximately $\alpha = 0.2$. But if we assume that $\kappa^2 \mathbb{V}^{(4)}_{6,24}$ should be between $\kappa^2 \mathbb{V}^{(4)}_{6,14}$ and $\kappa^2 \mathbb{V}^{(4)}_{6,16}$, we should rather take $\alpha \approx 0.25$. So we take $\alpha = 0.25$. The extrapolation for $L = 6$ with $Q = 14$ is then $\kappa^2 \mathbb{V}^{(4)}_{6,24} \approx -0.0544$. For $L = 8$ and $L = 10$ we take $Q = 10$ and we find $\kappa^2 \mathbb{V}^{(4)}_{8,32} \approx -0.0514$ and $\kappa^2 \mathbb{V}^{(4)}_{10,40} \approx -0.0513$. As a check that α doesn't depend much on Q, taking $Q = 10$ for $L = 4$ would give $\kappa^2 \mathbb{V}^{(4)}_{4,16} \approx -0.05426$, not terribly bad. We list the values of $\kappa^2 \mathbb{V}^{(4)}_{L,4L}$ with three significant digits, in the last line of table 5.

Now we would like to make a final extrapolation to estimate $\kappa^2 \mathbb{V}^{(4)}_{L,4L}$ as $L \to \infty$. Fits of the form

$$\kappa^2 \mathbb{V}^{(4)}_{L,4L} = f_0 + \frac{f_1}{L^\gamma} \tag{3.12}$$

are in general working quite well in open as well as closed string field theory. The exponent γ, usually an integer or half-integer, must be guessed in some way, more or less heuristically. Since our values for $L = 4, 6$ and $L = 8, 10$ are very similar, we feed the fit with only the values at $L = 2, 6, 10$. Leaving γ free, we find that these three values are perfectly fitted with $\gamma = 1.76$, and we take this as indication that we should take $\gamma = 2$. With this last fit we find, with two significant digits

$$\boxed{\lim_{L\to\infty} \kappa^2 \mathbb{V}^{(4)}_{L,4L} \approx -0.050}\,. \tag{3.13}$$

Although it is harder to make an extrapolation from the data of the scheme A (table 4), the value (3.13) fits well with it, in particular it is between the last two values.

We can do similar extrapolations of the vacuum expectation values of the tachyon and dilaton. For the tachyon we obtain an oscillation pattern very similar to the one of the potential value, and we find

$$t \approx 0.29\,. \tag{3.14}$$

The values for the dilaton, however, do not follow the same oscillating pattern and we are not able to evaluate a reliable extrapolation for $L > 4$. At $L = 2$ and $L = 4$ we find $d = 0.439$ and $d = 0.435$ respectively. Our best estimation based on those two values is thus

$$d \approx 0.43\,. \tag{3.15}$$

These values are again compatible with the data from scheme A.

4. Conclusions and prospects

In this paper we have considered nonpolynomial closed string field theory truncated at polynomial order four. We have then truncated the string field to level ten and have studied the nonperturbative minimum of the potential. In [5], an investigation of the low-energy effective action of the tachyon, dilaton and graviton of closed bosonic string theory led to the suggestion that if CSFT has a nonperturbative minimum, its action density should vanish. The results of the present paper do not support this supposition at quartic order. Instead, we find that the quartic potential has a minimum with height -0.050.

Potential	t	d	f_1	f_2	f_3	g_1	Value of the potential
$\mathbb{V}^{(4,t^5)}_{10,0}$	0.3321	0	−0.03949	−0.005976	−0.01620	0	−0.05094
$\mathbb{V}^{(4,t^5)}_{10,2}$	0.2612	0.2650	−0.03506	−0.003927	−0.01285	−0.04436	−0.03380
$\mathbb{V}^{(4,t^5)}_{10,4}$	0.2187	0.3460	−0.03135	−0.001509	−0.009119	−0.05061	−0.02630
$\mathbb{V}^{(4,t^5)}_{10,6}$	0.2666	0.2156	−0.04480	−0.003522	−0.01353	−0.01968	−0.03370
$\mathbb{V}^{(4,t^5)}_{10,8}$	0.2599	0.2359	−0.05041	−0.004857	−0.01657	−0.03693	−0.03276
$\mathbb{V}^{(4,t^5)}_{10,10}$	0.2570	0.2479	−0.04777	−0.004227	−0.01562	−0.03966	−0.03243

Table 6: The results of the truncation scheme A with the term t^5 included.

The question that we can ask now, is how the result (3.13) changes as we include higher order terms in the action (i.e. quintic term, sixtic term, etc. . .). In a separate paper [13] one of us has computed the five-tachyon contact term. Other quintic terms of higher level will follow [14], but we want here to already see how the results change if we include the t^5 term in the potential. In our normalization we have [13]

$$\kappa^2 V^{(5)}_{0,0} = 9.924\, t^5 \,.$$

Since the tachyon expectation value is positive at the vacuum, we expect this term to increase the value of the potential at the minimum. We make the definition

$$\mathbb{V}^{(4,t^5)}_{L,M} \equiv \mathbb{V}^{(4)}_{L,M} + V^{(5)}_{0,0} \,,$$

and repeat our analysis in the truncation scheme A. We find, as expected, that all values of the potential are shallower. But we also note that the oscillations are less strong than in table 4; that might be a sign that the results of level truncation will be improved when we include the quintic term, and that this procedure of truncating the action order by order is convergent. We emphasize however that quintic terms of higher level are necessary to reach any conclusion.

The conclusion that we can make at this point, is that at quartic order, the vacuum has a nonzero depth. It is possible that the higher orders contributions are important enough to make this depth converge to zero. It is also possible that the vacuum has a nonzero depth, close to what we find at quartic order. In this last case, it will be very interesting to try to understand what is this vacuum. Hopefully, the upcoming calculation at quintic order will make it possible to decide which one of the two alternatives is the right one.

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A. The quadratic and cubic potentials with fields of level up to six

In this appendix we want to write the potential $\mathbb{V}^{(3)}_{L,3L}$ with the fields level $L = 6$. It is decomposed in terms of quadratic potentials $V^{(2)}_M$ and cubic potentials $V^{(3)}_M$ at level M.

$$\mathbb{V}^{(3)}_{6,18} = V^{(2)}_0 + V^{(2)}_8 + V^{(2)}_{12} + V^{(3)}_0 + V^{(3)}_4 + V^{(3)}_6 + V^{(3)}_8 + V^{(3)}_{10} + V^{(3)}_{12} + V^{(3)}_{14} + V^{(3)}_{16} + V^{(3)}_{18}\,. \quad \text{(A.1)}$$

For the quadratic potentials we have

$$\begin{aligned}
\kappa^2 V^{(2)}_0 &= -t^2 && \text{(A.2)}\\
\kappa^2 V^{(2)}_8 &= f_1^2 + 169 f_2^2 - 26 f_3^2 - 2 g_1^2 && \text{(A.3)}\\
\kappa^2 V^{(2)}_{12} &= 4\psi_7^2 - 676\psi_9^2 - 4\psi_{10}^2 + 5408\psi_{13}^2 + 4\psi_{16}^2 - 104\psi_8\psi_{11} + 4\psi_{12}\psi_{14} + 416\psi_{15}\psi_{17}\,. && \text{(A.4)}
\end{aligned}$$

And the cubic potentials are

$$\begin{aligned}
\kappa^2 V^{(3)}_0 &= \frac{6561 t^3}{4096} && \text{(A.5)}\\
\kappa^2 V^{(3)}_4 &= -\frac{27 t d^2}{32} + \frac{3267 f_1 t^2}{4096} + \frac{114075 f_2 t^2}{4096} - \frac{19305 f_3 t^2}{2048} && \text{(A.6)}\\
\kappa^2 V^{(3)}_6 &= -\frac{25}{8} d g_1 t && \text{(A.7)}\\
\kappa^2 V^{(3)}_8 &= -\frac{f_1 d^2}{96} - \frac{4225 f_2 d^2}{864} + \frac{65 f_3 d^2}{144} + \frac{325}{432} t\psi_8 d - \frac{4225}{432} t\psi_9 d - \frac{25}{144} t\psi_{10} d + \frac{325}{144} t\psi_{11} d + \frac{361 f_1^2 t}{12288}\\
&\quad + \frac{57047809 f_2^2 t}{110592} + \frac{470873 f_3^2 t}{27648} - \frac{49 g_1^2 t}{24} + \frac{511225 f_1 f_2 t}{55296} - \frac{13585 f_1 f_3 t}{9216} - \frac{5400395 f_2 f_3 t}{27648} && \text{(A.8)}\\
\kappa^2 V^{(3)}_{10} &= -\frac{400}{729}\psi_7 d^2 + \frac{50}{729}\psi_{12} d^2 + \frac{200}{729}\psi_{14} d^2 + \frac{200}{729}\psi_{16} d^2 - \frac{9025 f_1 g_1 d}{5832} - \frac{105625 f_2 g_1 d}{5832} + \frac{30875 f_3 g_1 d}{2916}\\
&\quad + \frac{6175 g_1 t\psi_8}{1944} - \frac{105625 g_1 t\psi_9}{5832} - \frac{361}{216} g_1 t\psi_{10} + \frac{6175}{648} g_1 t\psi_{11} + \frac{50}{729} f_1 t\psi_{12} + \frac{346112}{729} f_2 t\psi_{13} + \frac{200}{729} f_1 t\psi_{14}\\
&\quad - \frac{8320}{729} f_3 t\psi_{15} - \frac{200}{729} f_1 t\psi_{16} + \frac{16640}{729} f_3 t\psi_{17} && \text{(A.9)}\\
\kappa^2 V^{(3)}_{12} &= \frac{f_1^3}{4096} + \frac{1525225 f_2 f_1^2}{8957952} - \frac{1235 f_3 f_1^2}{55296} + \frac{6902784889 f_2^2 f_1}{80621568} + \frac{1884233 f_3^2 f_1}{2239488} - \frac{961 g_1^2 f_1}{157464} - \frac{102607505 f_2 f_3 f_1}{6718464}\\
&\quad + \frac{325 d\psi_8 f_1}{34992} - \frac{4225 d\psi_9 f_1}{34992} - \frac{25 d\psi_{10} f_1}{11664} + \frac{325 d\psi_{11} f_1}{11664} + \frac{74181603769 f_2^3}{26873856} - \frac{31167227 f_3^3}{3359232}\\
&\quad + \frac{4965049817 f_2 f_3^2}{20155392} - \frac{207025 f_2 g_1^2}{17496} + \frac{14105 f_3 g_1^2}{26244} + \frac{128 t\psi_7^2}{19683} - \frac{105625 t\psi_8^2}{629856} - \frac{57047809 t\psi_9^2}{629856}\\
&\quad - \frac{1207801 t\psi_{10}^2}{629856} - \frac{105625 t\psi_{11}^2}{69984} + \frac{625 t\psi_{12}^2}{19683} + \frac{44302336 t\psi_{13}^2}{19683} + \frac{10000 t\psi_{14}^2}{19683} - \frac{332800 t\psi_{15}^2}{19683} + \frac{8528 t\psi_{16}^2}{19683}\\
&\quad - \frac{1331200 t\psi_{17}^2}{19683} - \frac{22628735129 f_2^2 f_3}{13436928} - \frac{33856 d g_1\psi_7}{19683} + \frac{2454725 d f_2\psi_8}{314928} - \frac{9815 d f_3\psi_8}{17496} - \frac{57047809 d f_2\psi_9}{314928}\\
&\quad + \frac{490945 d f_3\psi_9}{52488} + \frac{2454725 t\psi_8\psi_9}{314928} - \frac{105625 d f_2\psi_{10}}{104976} + \frac{1625 d f_3\psi_{10}}{17496} + \frac{357175 t\psi_8\psi_{10}}{314928} - \frac{105625 t\psi_9\psi_{10}}{104976}\\
&\quad + \frac{2454725 d f_2\psi_{11}}{104976} - \frac{9815 d f_3\psi_{11}}{5832} - \frac{8300747 t\psi_8\psi_{11}}{314928} + \frac{2454725 t\psi_9\psi_{11}}{104976} + \frac{357175 t\psi_{10}\psi_{11}}{104976} + \frac{1400 d g_1\psi_{12}}{6561}\\
&\quad + \frac{5600 d g_1\psi_{14}}{6561} + \frac{392 t\psi_{12}\psi_{14}}{2187} + \frac{17056 d g_1\psi_{16}}{19683} - \frac{1400 t\psi_{12}\psi_{16}}{6561} - \frac{5600 t\psi_{14}\psi_{16}}{6561} + \frac{372736 t\psi_{15}\psi_{17}}{6561} && \text{(A.10)}\\
\kappa^2 V^{(3)}_{14} &= -\frac{211250\psi_{12} f_3^2}{531441} - \frac{41879552\psi_{13} f_3^2}{531441} - \frac{845000\psi_{14} f_3^2}{531441} + \frac{5948800\psi_{15} f_3^2}{531441} + \frac{845000\psi_{16} f_3^2}{531441}\\
&\quad - \frac{11897600\psi_{17} f_3^2}{531441} - \frac{27055015 g_1\psi_8 f_3}{2125764} + \frac{233198875 g_1\psi_9 f_3}{2125764} + \frac{1021345 g_1\psi_{10} f_3}{236196} - \frac{27055015 g_1\psi_{11} f_3}{708588}\\
&\quad - \frac{78400 g_1^2\psi_7}{59049} + \frac{5106725 f_1 g_1\psi_8}{4251528} + \frac{46639775 f_2 g_1\psi_8}{1417176} + \frac{426400 d\psi_7\psi_8}{531441} - \frac{38130625 f_1 g_1\psi_9}{4251528}\\
&\quad - \frac{1426195225 f_2 g_1\psi_9}{4251528} - \frac{3380000 d\psi_7\psi_9}{531441} - \frac{683929 f_1 g_1\psi_{10}}{1417176} - \frac{1525225 f_2 g_1\psi_{10}}{157464} - \frac{53792 d\psi_7\psi_{10}}{177147}\\
&\quad + \frac{5106725 f_1 g_1\psi_{11}}{1417176} + \frac{46639775 f_2 g_1\psi_{11}}{472392} + \frac{426400 d\psi_7\psi_{11}}{177147} + \frac{9800 g_1^2\psi_{12}}{59049} + \frac{211250 f_1 f_2\psi_{12}}{531441}\\
&\quad - \frac{32500 d\psi_8\psi_{12}}{531441} + \frac{422500 d\psi_9\psi_{12}}{531441} + \frac{1900 d\psi_{10}\psi_{12}}{59049} - \frac{24700 d\psi_{11}\psi_{12}}{59049} + \frac{41879552 f_1 f_2\psi_{13}}{531441}
\end{aligned}$$

$$
\begin{aligned}
& -\frac{44302336d\psi_9\psi_{13}}{531441}+\frac{39200g_1^2\psi_{14}}{59049}+\frac{845000f_1f_2\psi_{14}}{531441}-\frac{98800d\psi_8\psi_{14}}{177147}+\frac{1690000d\psi_9\psi_{14}}{531441}\\
& +\frac{7600d\psi_{10}\psi_{14}}{59049}-\frac{130000d\psi_{11}\psi_{14}}{177147}-\frac{5948800f_1f_2\psi_{15}}{531441}-\frac{332800d\psi_8\psi_{15}}{531441}+\frac{252928d\psi_{11}\psi_{15}}{59049}\\
& +\frac{39200g_1^2\psi_{16}}{59049}-\frac{845000f_1f_2\psi_{16}}{531441}-\frac{213200d\psi_8\psi_{16}}{531441}+\frac{1690000d\psi_9\psi_{16}}{531441}+\frac{30992d\psi_{10}\psi_{16}}{177147}\\
& -\frac{213200d\psi_{11}\psi_{16}}{177147}+\frac{11897600f_1f_2\psi_{17}}{531441}-\frac{505856d\psi_8\psi_{17}}{177147}+\frac{665600d\psi_{11}\psi_{17}}{177147}
\end{aligned}
\tag{A.11}
$$

$$
\begin{aligned}
\kappa^2V_{16}^{(3)} = & \frac{5274752f_1\psi_7^2}{14348907}+\frac{540800f_2\psi_7^2}{14348907}-\frac{3377920f_3\psi_7^2}{14348907}+\frac{6219200g_1\psi_8\psi_7}{4782969}-\frac{143041600g_1\psi_9\psi_7}{14348907}\\
& -\frac{270400g_1\psi_{10}\psi_7}{531441}+\frac{6219200g_1\psi_{11}\psi_7}{1594323}-\frac{105625f_1\psi_8^2}{51018336}-\frac{1426195225f_2\psi_8^2}{459165024}+\frac{12273625f_3\psi_8^2}{76527504}\\
& -\frac{57047809f_1\psi_9^2}{51018336}-\frac{74181603769f_2\psi_9^2}{51018336}+\frac{2057157739f_3\psi_9^2}{25509168}-\frac{131997121f_1\psi_{10}^2}{459165024}-\frac{5102959225f_2\psi_{10}^2}{459165024}\\
& +\frac{820716715f_3\psi_{10}^2}{229582512}-\frac{105625f_1\psi_{11}^2}{5668704}-\frac{1426195225f_2\psi_{11}^2}{51018336}+\frac{12273625f_3\psi_{11}^2}{8503056}+\frac{625f_1\psi_{12}^2}{19683}\\
& +\frac{2640625f_2\psi_{12}^2}{14348907}-\frac{81250f_3\psi_{12}^2}{531441}+\frac{5360582656f_1\psi_{13}^2}{14348907}+\frac{15993143296f_2\psi_{13}^2}{1594323}-\frac{18518376448f_3\psi_{13}^2}{4782969}\\
& +\frac{10000f_1\psi_{14}^2}{19683}+\frac{42250000f_2\psi_{14}^2}{14348907}-\frac{1300000f_3\psi_{14}^2}{531441}-\frac{3660800f_1\psi_{15}^2}{531441}-\frac{411008000f_2\psi_{15}^2}{4782969}\\
& +\frac{750131200f_3\psi_{15}^2}{14348907}+\frac{3795152f_1\psi_{16}^2}{14348907}+\frac{36030800f_2\psi_{16}^2}{14348907}-\frac{4852640f_3\psi_{16}^2}{4782969}-\frac{14643200f_1\psi_{17}^2}{531441}\\
& -\frac{1644032000f_2\psi_{17}^2}{4782969}+\frac{3000524800f_3\psi_{17}^2}{14348907}+\frac{2454725f_1\psi_8\psi_9}{25509168}+\frac{10285788695f_2\psi_8\psi_9}{76527504}\\
& -\frac{275396485f_3\psi_8\psi_9}{38263752}+\frac{3733925f_1\psi_8\psi_{10}}{76527504}+\frac{2697742775f_2\psi_8\psi_{10}}{229582512}-\frac{251765605f_3\psi_8\psi_{10}}{114791256}\\
& -\frac{105625f_1\psi_9\psi_{10}}{8503056}-\frac{1426195225f_2\psi_9\psi_{10}}{76527504}+\frac{12273625f_3\psi_9\psi_{10}}{12754584}-\frac{86776417f_1\psi_8\psi_{11}}{76527504}\\
& -\frac{19456250905f_2\psi_8\psi_{11}}{76527504}+\frac{1834363531f_3\psi_8\psi_{11}}{38263752}+\frac{2454725f_1\psi_9\psi_{11}}{8503056}+\frac{10285788695f_2\psi_9\psi_{11}}{25509168}\\
& -\frac{275396485f_3\psi_9\psi_{11}}{12754584}+\frac{3733925f_1\psi_{10}\psi_{11}}{25509168}+\frac{2697742775f_2\psi_{10}\psi_{11}}{76527504}-\frac{251765605f_3\psi_{10}\psi_{11}}{38263752}\\
& -\frac{455000g_1\psi_8\psi_{12}}{4782969}+\frac{5915000g_1\psi_9\psi_{12}}{4782969}+\frac{26600g_1\psi_{10}\psi_{12}}{531441}-\frac{345800g_1\psi_{11}\psi_{12}}{531441}-\frac{2215116800g_1\psi_9\psi_{13}}{14348907}\\
& -\frac{1383200g_1\psi_8\psi_{14}}{1594323}+\frac{23660000g_1\psi_9\psi_{14}}{4782969}+\frac{106400g_1\psi_{10}\psi_{14}}{531441}-\frac{1820000g_1\psi_{11}\psi_{14}}{1594323}+\frac{150152f_1\psi_{12}\psi_{14}}{14348907}\\
& +\frac{1656200f_2\psi_{12}\psi_{14}}{1594323}+\frac{997360f_3\psi_{12}\psi_{14}}{4782969}+\frac{4659200g_1\psi_8\psi_{15}}{4782969}+\frac{34611200g_1\psi_9\psi_{15}}{14348907}-\frac{12646400g_1\psi_{11}\psi_{15}}{1594323}\\
& -\frac{3234400g_1\psi_8\psi_{16}}{4782969}+\frac{72061600g_1\psi_9\psi_{16}}{14348907}+\frac{164000g_1\psi_{10}\psi_{16}}{531441}-\frac{3234400g_1\psi_{11}\psi_{16}}{1594323}+\frac{27400f_1\psi_{12}\psi_{16}}{531441}\\
& -\frac{5915000f_2\psi_{12}\psi_{16}}{4782969}+\frac{5590000f_3\psi_{12}\psi_{16}}{14348907}+\frac{109600f_1\psi_{14}\psi_{16}}{531441}-\frac{23660000f_2\psi_{14}\psi_{16}}{4782969}\\
& +\frac{22360000f_3\psi_{14}\psi_{16}}{14348907}+\frac{25292800g_1\psi_8\psi_{17}}{4782969}-\frac{69222400g_1\psi_9\psi_{17}}{14348907}-\frac{9318400g_1\psi_{11}\psi_{17}}{1594323}\\
& -\frac{80244736f_1\psi_{15}\psi_{17}}{14348907}+\frac{460328960f_2\psi_{15}\psi_{17}}{1594323}-\frac{127901696f_3\psi_{15}\psi_{17}}{4782969}
\end{aligned}
\tag{A.12}
$$

$$
\begin{aligned}
\kappa^2V_{18}^{(3)} = & -\frac{99123200\psi_{12}\psi_7^2}{387420489}-\frac{396492800\psi_{14}\psi_7^2}{387420489}-\frac{396492800\psi_{16}\psi_7^2}{387420489}-\frac{57402800\psi_8^2\psi_7}{129140163}-\frac{22819123600\psi_9^2\psi_7}{387420489}\\
& -\frac{144400\psi_{10}^2\psi_7}{4782969}-\frac{57402800\psi_{11}^2\psi_7}{14348907}+\frac{3220599200\psi_8\psi_9\psi_7}{387420489}+\frac{8101600\psi_8\psi_{10}\psi_7}{43046721}-\frac{227271200\psi_9\psi_{10}\psi_7}{129140163}\\
& -\frac{227271200\psi_8\psi_{11}\psi_7}{129140163}+\frac{3220599200\psi_9\psi_{11}\psi_7}{129140163}+\frac{8101600\psi_{10}\psi_{11}\psi_7}{14348907}+\frac{5281250\psi_8^2\psi_{12}}{387420489}\\
& +\frac{2852390450\psi_9^2\psi_{12}}{387420489}+\frac{18050\psi_{10}^2\psi_{12}}{4782969}+\frac{3050450\psi_{11}^2\psi_{12}}{4782969}-\frac{245472500\psi_8\psi_9\psi_{12}}{387420489}-\frac{617500\psi_8\psi_{10}\psi_{12}}{43046721}\\
& +\frac{8027500\psi_9\psi_{10}\psi_{12}}{43046721}+\frac{14350700\psi_8\psi_{11}\psi_{12}}{43046721}-\frac{186559100\psi_9\psi_{11}\psi_{12}}{43046721}-\frac{469300\psi_{10}\psi_{11}\psi_{12}}{4782969}\\
& +\frac{553779200\psi_8^2\psi_{13}}{387420489}+\frac{553779200\psi_{11}^2\psi_{13}}{43046721}-\frac{1107558400\psi_9\psi_{10}\psi_{13}}{129140163}+\frac{12201800\psi_8^2\psi_{14}}{43046721}\\
& +\frac{11409561800\psi_9^2\psi_{14}}{387420489}+\frac{72200\psi_{10}^2\psi_{14}}{4782969}+\frac{21125000\psi_{11}^2\psi_{14}}{43046721}-\frac{746236400\psi_8\psi_9\psi_{14}}{129140163}-\frac{1877200\psi_8\psi_{10}\psi_{14}}{14348907}\\
& +\frac{32110000\psi_9\psi_{10}\psi_{14}}{43046721}+\frac{57402800\psi_8\psi_{11}\psi_{14}}{43046721}-\frac{981890000\psi_9\psi_{11}\psi_{14}}{129140163}-\frac{2470000\psi_{10}\psi_{11}\psi_{14}}{14348907}\\
& +\frac{108160000\psi_8^2\psi_{15}}{387420489}-\frac{82201600\psi_{11}^2\psi_{15}}{14348907}-\frac{2513638400\psi_8\psi_9\psi_{15}}{387420489}-\frac{6323200\psi_8\psi_{10}\psi_{15}}{43046721}\\
& +\frac{138444800\psi_9\psi_{10}\psi_{15}}{129140163}+\frac{2513638400\psi_9\psi_{11}\psi_{15}}{129140163}+\frac{6323200\psi_{10}\psi_{11}\psi_{15}}{14348907}+\frac{16055000\psi_8^2\psi_{16}}{129140163}
\end{aligned}
$$

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$$
\begin{aligned}
&+\frac{11409561800\psi_9^2\psi_{16}}{387420489}+\frac{72200\psi_{10}^2\psi_{16}}{4782969}+\frac{16055000\psi_{11}^2\psi_{16}}{14348907}-\frac{1610299600\psi_8\psi_9\psi_{16}}{387420489}-\frac{4050800\psi_8\psi_{10}\psi_{16}}{43046721}\\
&+\frac{130941200\psi_9\psi_{10}\psi_{16}}{129140163}+\frac{57402800\psi_8\psi_{11}\psi_{16}}{43046721}-\frac{1610299600\psi_9\psi_{11}\psi_{16}}{129140163}-\frac{4050800\psi_{10}\psi_{11}\psi_{16}}{14348907}\\
&+\frac{164403200\psi_8^2\psi_{17}}{129140163}-\frac{216320000\psi_{11}^2\psi_{17}}{43046721}-\frac{5027276800\psi_8\psi_9\psi_{17}}{387420489}-\frac{12646400\psi_8\psi_{10}\psi_{17}}{43046721}\\
&-\frac{276889600\psi_9\psi_{10}\psi_{17}}{129140163}+\frac{5027276800\psi_9\psi_{11}\psi_{17}}{129140163}+\frac{12646400\psi_{10}\psi_{11}\psi_{17}}{14348907}\,.
\end{aligned}
\tag{A.13}
$$

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